

Isgur-Wise Function and Observables of Λ_b Baryon

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Abstract

In connection with planned experiments devoted to investigation of semileptonic decays of beauty baryons Isgur-Wise function and observables of Λ_b baryon (decay rates and distributions, leptonic spectra and asymmetry parameters) are calculated in the framework of diquark model with taking into account of infrared regime for heavy quark.

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1 Introduction

Weak decays of heavy hadrons containing a single heavy quark can be considered as a unique tool to determine Cabibbo-Kabayashi-Maskawa matrix elements, original source to probe hadronic structure and investigation of effects beyond Standard Model.

The growth of interest to such processes is connected with considerable progress in experimental investigation and new theoretical ideas which were borned. One has to remark that during a long time the experimental programs were concentrated mainly on heavy mesons and charm baryons. Nowadays the facilities of modern accelerators allow to us investigate properties of beauty baryons. Particular first observation of Λ_b baryon was made in the decay $\Lambda_b \rightarrow J/\Psi \Lambda$ at the $p\bar{p}$ collider (CERN) [1]. Semileptonic decays of Λ_b baryon $\Lambda_b \rightarrow \Lambda_c X e \nu$ were studied by ALEPH and OPAL group at LEP [2, 3]. Observation of Λ_b production in the Z^0 decays and measuring of Λ_b lifetime were fulfilled by DELPHI Collaboration [4].

From a theoretical point of view weak decays of heavy quarks are interested because of a new type of symmetry of strong interactions was discovered - spin-flavour symmetry in the world of heavy quarks (Isgur-Wise symmetry) [5] and also heavy quark effective theory (HQET) [5]-[7] - perturbative calculating scheme for investigation properties of hadrons containing a single heavy quark. Isgur-Wise symmetry manifested itself in the limit when heavy quark masses go to infinity $m_Q \rightarrow \infty, Q = b, c$ (Isgur-Wise limit). The consequences of these symmetries for weak heavy-hadron form factors were worked out by Isgur and Wise in ref. [5, 6]. Isgur and Wise showed [5], that form factors of B-meson weak decays $B \rightarrow D \ell \nu$ $B \rightarrow D^* \ell \nu$ are expressed through universal function $\xi(\omega)$ (mesonic Isgur-Wise function), where ω is the scalar product of the four-velocities of the initial and final heavy-hadron states.

In paper [6] the consequences of Isgur-Wise symmetry were obtained for form factors of semileptonic decays of heavy baryons. It was shown that in the Isgur-Wise limit (baryonic form factors satisfied to group conditions and expressed through three unknow universal functions $\zeta(\omega)$, $\eta(\omega)$ $\iota(\omega)$ [6]. For example, matrix element corresponding to decay $\Lambda_b \rightarrow \Lambda_c e \nu$ in the Isgur-Wise limit has a trivial shape $J_\mu(v, v') \propto \zeta(\omega) \bar{u}(v') \gamma_\mu (1 + \gamma_5) u(v)$.

For decays $\Omega_b(\Sigma_b) \rightarrow \Omega_c(\Sigma_c)e\nu$ $\Omega_b(\Sigma_b) \rightarrow \Omega_c^*(\Sigma^*)e\nu$ connections between form factors are more complicated. However, all form factors can be expressed through two functions $\eta(\omega)$ and $\iota(\omega)$. Unfortunately, HQET as realistic theory of heavy flavoured hadrons has no possibility to calculate functions $\zeta(\omega)$, $\eta(\omega)$ and $\iota(\omega)$. So calculation of barionic Isgur-Wise functions got dissemination within various phenomenological approaches: Infinite Momentum Frame (IMF) models [8, 9], QCD Sum Rules [10], Quark Confinement Models [11] and so on.

In [8, 9] semileptonic decay $\Lambda_b \rightarrow \Lambda_c e \nu$ was considered in the framework of IMF model where it was assumed that heavy baryon consists of a heavy quark and light spin-zero diquark system. In ref. [9] Bauer-Stech-Wirbel type infinite momentum frame wave functions for the heavy Λ -type baryons were used and in [8] Drell-Yan ones. Isgur-Wise function $\zeta(\omega)$ [8, 9] and various observables of $\Lambda_b \rightarrow \Lambda_c e \nu$ decay (rates, spectra and asymmetry parameters) were computed. Using QCD Sum Rules technique $\zeta(\omega)$ form factor was calculated in paper [10]. In [11] semileptonic decays of heavy baryons $\Lambda_b \rightarrow \Lambda_c e \nu$ and $\Sigma_b \rightarrow \Sigma_c e \nu$ are considered in the framework of Quark Confinement Model [12] with taking into account of so-called *quark-diquark approximation* [11]. It was showed in [11, 12] that all form factors of $\Lambda_b \rightarrow \Lambda_c e \nu$ and $\Sigma_b \rightarrow \Sigma_c e \nu$ decay are expressed through a single universal function $\Phi(\omega)$ which is equal identically to Isgur-Wise function ζ and is given the expression $\Phi(\omega) \equiv \zeta(\omega) = \ln(\omega + \sqrt{\omega^2 - 1})(\omega^2 - 1)^{-1/2}$. Later it was founded by Xu [13] that obtained results within QCM [11] contradict to Bjorken sum rule for semileptonic Ω_b decay which give the following restriction on function $\Phi(\omega)$: $\Phi^2(\omega) \leq 3/(1 + 2\omega^2)$. So form factors obtained in the QCM are sufficiently "hard".

In our paper we obtained suppression of baryonic Isgur-Wise functions using for heavy quark propagator so-called "infrapropagator", which describes behaviour of heavy quark near its mass-shell, i.e. in the limit when heavy quark mass goes to infinity (Isgur-Wise limit). Recently, in [14, 15] idea about infrared behaviour of heavy quark in the limit $m_Q \rightarrow \infty$ was successfully used in calculation of mesonic Isgur-Wise function $\xi(\omega)$. Then we use obtained Isgur-Wise function in the calculation of observables of Λ_b baryon (decay rates and distributions, leptonic spectra and asymmetry parameters).

2 Model

At the consideration of $b \rightarrow c$ decays of heavy baryons we will use *diquark factorization of baryon structure*, which is based on a observation that light degrees of freedom in these processes are manifested itself as spectator or diquark. Thus heavy baryon can be presented as two-particle bound state of heavy quark and light diquark. In the case of Λ_b and Λ baryons light diquark D^a must possess by the following quantum numbers: spin $J = 0$, isospin $I = 0$ and positive P-parity. Hence quark-diquark current with quantum numbers of Λ_b and Λ baryons has the form $J_Q(x) = Q^a(x)D^a(x)$, $Q = c, b$. The interaction Lagrangian describing transition of heavy baryon Λ_Q into quark-diquark pair and *vice versa* is written as

$$\mathcal{L}_Q(x) = g_{\Lambda_Q} \bar{\Lambda}_Q J_Q + ..$$

Full Lagrangian needed for describing of decay $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ is given by the formula:

$$\mathcal{L}_{full} = \sum_Q \mathcal{L}_Q + \mathcal{L}_{weak} + .., \quad \mathcal{L}_{weak} = \frac{G_F}{\sqrt{2}} \ell^\mu \bar{c}^a V_{bc} O_\mu b^a \quad (1)$$

where V_{bc} is Kabayashi-Maskava matrix element.

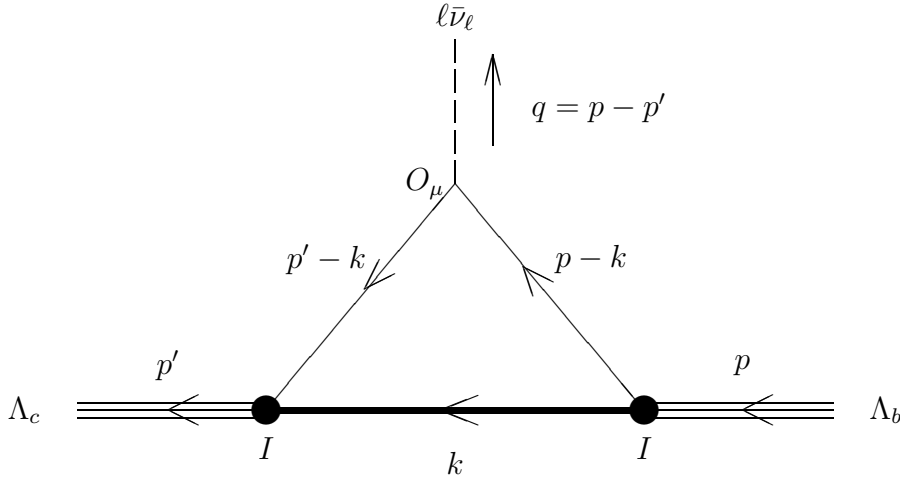


Fig.1 Semileptonic decay of Λ_b baryon.

The corresponding Feynman diagram is drawn in Fig.1. As light diquark propagator we will use standard propagator of scalar field $S_D(p^2) = 1/(M_D^2 - p^2)$ where M_D is a diquark mass.

As it is known from HQET [5]-[7] in the Isgur-Wise limit heavy quark is near its mass-shell, i.e. infrared regime comes for heavy quark. Infrared asymptotics of one-particle Green function was investigated in an Abelian theory (Quantum Electrodynamics) in many papers (see, for example, ref. [16]). So-called *infrapropagator* of electron has the following shape

$$G(p, \nu) = (m - \not{p})^{-1} s(p^2, \nu), \quad s(p^2, \nu) = (1 - p^2/m^2)^{-\nu} \quad (2)$$

Here parameter ν is connected with gauge parameter d_ℓ by well-known condition

$$\nu = (\alpha_{em}/4\pi)(3 - d_\ell), \quad (3)$$

where α_{em} is a fine structure constant.

In the papers [14, 15] as the first approximation *infrapropagator* of an Abelian theory is used in the calculations of Isgur-Wise function. We also will use this propagator. Particular heavy quark propagator is given by formula

$$S_Q(p, \nu) = \frac{m_Q + \not{p}}{m_Q^2} \left(\frac{1}{1 - p^2/m_Q^2} \right)^{1+\nu} \quad (4)$$

The parameter ν must be bigger than zero to make all matrix elements ultraviolet finite. Therefore our model contains three parameters: light diquark mass M_D , parameter $\bar{\Lambda} = M_{B_Q} - m_{B_Q}$ and infrared parameter ν . Experimental restrictions on the parameter $\bar{\Lambda}$ are absent. Theoretical evaluation of value $\bar{\Lambda}$ was made using QCD Sum Rules Technique by Neubert [17]. It was founded that $\bar{\Lambda} = 0.50 \pm 0.07$ GeV. Within QCM under consideration of leptonic decays of B and D mesons it was obtained that the parameter $\bar{\Lambda}$ must exchange in limits $0 \leq \bar{\Lambda} \leq 0.6$ GeV. In our calculations we will fix the parameter $\bar{\Lambda}$ equaled to 0.6 GeV (as in various phenomenological approaches, see for example ref. [9]).

3 $\Lambda_b \rightarrow \Lambda_c$ Isgur-Wise Function

Isgur-Wise function in our model depends on parameter ν and ratio of diquark mass and parameter Λ : $R = M_D/\bar{\Lambda}$ and is written as

$$\zeta(\omega, R, \nu) = \frac{\Phi(\omega, R, \nu)}{\Phi(1, R, \nu)}, \quad \Phi(\omega, R, \nu) = \int_0^\infty du u^{1+2\nu} \int_0^1 \frac{dt}{\sqrt{1-t}} \frac{t^\nu}{[R^2 + (u-1)^2 + \frac{u^2 t}{2}(\omega-1)]^{1+2\nu}}$$

In our calculations of Isgur-Wise function we try to get the best agreement with results of IMF models [8, 9]. It was achieved when parameter ν is closer to value 1. Results for ω -dependence of Isgur-Wise function for parameter $\nu = 1$ and various meaning of parameter R in the interval $1.2 < R < 2$ are presented in Fig.2. For comparison results of other theoretical approaches are given (IMF model [9] and QCD Sum Rules [10]). One can see that a good agreement with IMF model is achieved when $R = 1.25$. This choice of parameter R will be called in our paper as *the best fit*. It is clear that in this case the diquark mass is equal to 0.7 GeV.

Let us to calculate also charge radius of Isgur-Wise function $\rho^2 = -d\zeta/d\omega|_{\omega=1}$

$$\rho^2 = \frac{(1+\nu)(1+2\nu)}{3+2\nu} \frac{\int_0^\infty du u^{3+2\nu} [R^2 + (u-1)^2]^{-2-2\nu}}{\int_0^\infty du u^{1+2\nu} [R^2 + (u-1)^2]^{-1-2\nu}} \quad (5)$$

The results for ρ^2 as function of $\bar{\Lambda}$ are given in Table 1.

Table 1. Charge radius of Isgur-Wise function.

$\bar{\Lambda}$ (GeV)	0.47	0.48	0.49	0.50	0.51	0.52	0.53
ρ^2	3.03	2.67	2.43	2.25	2.10	1.99	1.89

4 Weak Properties of Λ_b Baryon

Observables of semileptonic decays of Λ_b baryon (decay rates, differential distributions, leptonic spectra and asymmetry parameters) we will determine in the terms of helicity

amplitudes $H_{\lambda_f \lambda_W}^\Gamma$ [9, 18], where λ_f is helicity of baryon in the final state and λ_W is helicity of W-boson out of mass-shell. We will present the calculations of Λ_b baryon properties in the case of *the best fit*. In the Isgur-Wise limit helicity amplitudes are expressed through function $\zeta(\omega)$:

$$H_{\pm\frac{1}{2}0}^V = \zeta \sqrt{\frac{\omega - 1}{\omega_{max} - \omega}} (M_{\Lambda_b} + M_{\Lambda_c}), \quad H_{\pm\frac{1}{2}0}^A = \pm \zeta \sqrt{\frac{\omega + 1}{\omega_{max} + \omega}} (M_{\Lambda_b} - M_{\Lambda_c}),$$

$$H_{\pm\frac{1}{2}1}^V = -2\zeta \sqrt{M_{\Lambda_b} M_{\Lambda_c} (\omega - 1)}, \quad H_{\pm\frac{1}{2}1}^A = \mp 2\zeta \sqrt{M_{\Lambda_b} M_{\Lambda_c} (\omega + 1)}, \quad \omega_{max} = \frac{M_{\Lambda_b}^2 + M_{\Lambda_c}^2}{2M_{\Lambda_b} M_{\Lambda_c}}$$

Decay rates of semileptonic decays are calculated in accordance with formula

$$\Gamma = \int_1^{\omega_{max}} d\omega \frac{d\Gamma}{d\omega}, \quad \frac{d\Gamma}{d\omega} = \frac{d\Gamma_{T+}}{d\omega} + \frac{d\Gamma_{T-}}{d\omega} + \frac{d\Gamma_{L+}}{d\omega} + \frac{d\Gamma_{L-}}{d\omega} \quad (6)$$

where indices T and L denote partial contributions of the transverse ($\lambda_W = \pm 1$) and longitudinal ($\lambda_W = 0$) components of the current transition. Partial differential distributions are equal

$$\frac{d\Gamma_{T\pm}}{d\omega} = \kappa_\omega |H_{\pm\frac{1}{2}\pm 1}|^2, \quad \frac{d\Gamma_{L\pm}}{d\omega} = \kappa_\omega |H_{\pm\frac{1}{2}0}|^2, \quad \kappa_\omega = \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \frac{M_{\Lambda_c}^3}{6} (\omega_{max} - \omega) \sqrt{\omega^2 - 1} \quad (7)$$

where $H_{\lambda_f \lambda_W} = H_{\lambda_f \lambda_W}^V - H_{\lambda_f \lambda_W}^A$. The results for decay rates are given in Table 2. For comparison we present the results of IFM model [9]. The curves of differential distributions are drawn in Fig.3. One can underline that our results for $\frac{d\Gamma}{d\omega}$ are practically coincide with ones obtained in ref. [9].

Leptonic spectra $d\Gamma/dE_\ell$ is calculated by formula

$$\frac{d\Gamma}{dE_\ell} = \frac{d\Gamma_{T+}}{dE_\ell} + \frac{d\Gamma_{T-}}{dE_\ell} + \frac{d\Gamma_{L+}}{dE_\ell} + \frac{d\Gamma_{L-}}{dE_\ell} \quad (8)$$

Expressions for partial leptonic spectra are given by formulas

$$\frac{d\Gamma_{T\pm}}{dE_\ell} = \int_{\omega_{min}(E_\ell)}^{\omega_{max}} d\omega \kappa_E (1 \pm \cos \Theta)^2 |H_{\pm\frac{1}{2}\pm 1}|^2, \quad \frac{d\Gamma_{L\pm}}{dE_\ell} = \int_{\omega_{min}(E_\ell)}^{\omega_{max}} d\omega \kappa_E (1 - \cos^2 \Theta)^2 |H_{\pm\frac{1}{2}0}|^2,$$

$$\kappa_E = \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \frac{M_{\Lambda_c}^2}{8} (\omega_{max} - \omega), \quad \cos \Theta = \frac{E_\ell^{max} - 2E_\ell + M_{\Lambda_c}(\omega_{max} - \omega)}{M_{\Lambda_c} \sqrt{\omega^2 - 1}},$$

$$E_\ell^{max} = \frac{M_{\Lambda_b}^2 - M_{\Lambda_c}^2}{2M_{\Lambda_b}}, \quad \omega_{min}(E_\ell) = \omega_{max} - 2 \frac{E_\ell}{M_{\Lambda_c}} \frac{E_\ell^{max} - E_\ell}{M_{\Lambda_b} - 2E_\ell}$$

Our results for leptonic spectra are pictured on Fig.4.

Table 2. Decay rate of $\Lambda_b \rightarrow \Lambda_c \ell \nu_\ell$ decay (in units 10^{10} sec^{-1})

Model	Γ_{total}	Γ_T	Γ_{T+}	Γ_{T-}	Γ_L	Γ_{L+}	Γ_{L-}
QCM	4.07	1.74	0.47	1.27	2.32	0.11	2.21
IMF [9]	4.57	1.88	0.42	1.46	2.69	0.11	2.58

Let us to consider two-cascade weak decay $\Lambda_b \rightarrow \Lambda_c[\rightarrow \Lambda_s \pi] + W[\rightarrow \ell \nu_\ell]$ which is characterizes by asymmetry parameters. Formalism and detailed analysis of asymmetry parameters was fulfilled in paper [14]. Here we calculate asymmetry parameters of non-polarized Λ_b decay ($\alpha, \alpha', \alpha'', \gamma$) and polarized one (α_P, γ_P) which in the terms of helicity amplitudes are given by following expressions

$$\begin{aligned}
\alpha &= \frac{H_T^- + H_L^-}{H_T^+ + H_L^+}, \quad \alpha' = \frac{H_T^-}{H_T^+ + 2H_L^+}, \quad \alpha'' = \frac{H_T^+ - 2H_L^+}{H_T^+ + 2H_L^+}, \quad \gamma = \frac{2H_\gamma}{H_T^+ + H_L^+}, \\
\alpha_P &= \frac{H_T^- - H_L^-}{H_T^+ + H_L^+}, \quad \gamma_P = \frac{2H_{\gamma_P}}{H_T^+ + H_L^+}, \\
H_T^\pm &= |H_{1/2 \ 1}|^2 \pm |H_{-1/2 \ -1}|^2, \quad H_L^\pm = |H_{1/2 \ 0}|^2 \pm |H_{-1/2 \ 0}|^2 \\
H_\gamma &= \text{Re}(H_{-1/2 \ 0} H_{1/2 \ 1}^* + H_{1/2 \ 0} H_{-1/2 \ -1}^*) \quad H_{\gamma_P} = \text{Re}(H_{1/2 \ 0} H_{-1/2 \ 0}^*)
\end{aligned} \tag{9}$$

In our paper we will calculate average meanings of asymmetry parameters ($\langle \alpha \rangle, \langle \alpha' \rangle$ and so on) as result of separate integration over parameter ω of numerator and denominator of eq. (8) with weight $(\omega_{max} - \omega)\sqrt{\omega^2 - 1}$ in the interval $1 \leq \omega \leq \omega_{max}$. Results for average meanings are given in Table 3. Also the results of paper [9] are presented.

Table 3. Asymmetry parameters

Model	$\langle \alpha \rangle$	$\langle \alpha' \rangle$	$\langle \alpha'' \rangle$	$\langle \gamma \rangle$	$\langle \alpha_P \rangle$	$\langle \gamma_P \rangle$
QCM	-0.71	-0.13	-0.46	0.61	0.32	-0.19
IMF [9]	-0.71	-0.12	-0.46	0.61	0.33	-0.19

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